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# CAN ROBOT MANIPULATORS AVOID MOVING OBSTACLES IN 3D WORSPACES?

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***Abstract –*** *This paper presents a general methodology for the off-line three-dimensional optimal trajectory planning of robot manipulators in the presence of moving obstacles. Obstacle avoidance is obtained by adding penalty functions to the function to be minimized. Besides, constraints which describe minimal acceptable distance between potentially colliding parts are also included to the general non-linear optimization problem. The obstacles are protected by spherical or hyper-spherical security zones which are never penetrated by the end-effector. When dealing with moving obstacles either the objective function and the constraint functions have to be up-dated at each time instant. Multi-criterion objective functions are constructed to take into-account traveling time and mechanical energy which are both minimized simultaneously. Numerical applications show the efficiency of the methodology presented.*

***Key words:*** *Robot manipulator, Obstacle avoidance, Trajectory planning*

## 1. INTRODUCTION

Different engineering applications involve robot manipulators working in the presence of fixed or moving obstacles. When repetitive processes are concerned it is possible to develop a methodology to move a robot manipulator along a specified geometric path avoiding obstacles. In this case path planning is defined as finding continuos and mutually compatible trajectories for all its parts, such that the resulting motions is collision-free. This can be achieved for minimum cost configuration through optimization techniques.

The problem of optimization of the trajectory of robot manipulators in the presence of fixed obstacles has been treated by different authors (Ferreira & Sá da Costa, 1997). Previous papers presented the case in which the optimization problem was written as a multiobjective one, i. e., the total traveling time and the mechanical energy absorbed by the actuators were considered together to build a scalar objective function to be minimized (Richard, 1993) . When moving obstacles are sharing the same workspace occupied by the robot manipulator the optimization of the trajectory defined by the end-effector is complex ( Nearchou, 1998).

This complexity is associated with the large number of constraints to be taken into-account by the optimizer. These constraints are in this case time dependent.

This paper presents a design methodology to obtain the optimal off-line trajectory planning of robot manipulators when moving obstacles have to be avoided by the end-effector. The problem of optimal planning concerns the determination of the end-effector robot motion at a minimum time and minimum mechanical energy between two given points while satisfying the limits of the actuator efforts and avoiding collision with moving and fixed obstacles. The obstacle avoidance is expressed in terms of the distances between potentially colliding parts and the motion is represented using translation and rotational matrices. The dynamical model of the robot is derived using Euler-Lagrange's equations and Lagrange's energy function. The inertia terms of the actuators are not neglected and friction forces are included in the equations of motion. The joint trajectories are formulated using uniform cubic B-spline functions, given only the initial and final points. When obstacles are found in the three-dimensional workspace it is necessary to add penalty functions to the multi-objective problem to guarantee free-collision motion. The obstacles are protected by spherical or hyper-spherical security zones which are never penetrated by the end-effector. To reduce the computational effort only the closest obstacles are included in the penalty function at a given time. The important assumption is that complete information on the geometry of the robot and the obstacles are given beforehand. Two numerical applications related to a Stanford manipulator are presented focusing at the methodology developed in this paper. In both cases fixed and moving obstacles share the robot three-dimensional workspace.

## 2. PROBLEM STATEMENT

Let a manipulator with *n* degree of freedom (d.o.f.) , consider that *j* represents the joints and *m* represents the knots used to construct the trajectories. The task is to move the robot in the workspace avoiding the moving obstacles $\psi_l$ , while minimizing the traveling time and the energy consumed by the robot, subject to physical constraints and actuator limits.

The optimal traveling time and the minimum mechanical energy of the actuators are considered together to build a performance index such as the multi-criterion optimization problem is defined as follows:

Minimize:

$$F_c = \alpha_1 T + \alpha_2 \int_0^T \sum_{i=1}^{n} \left(u_i(t)\right)^2 dt + \alpha_3 \, f_{dis} \tag{1}$$

where: $T$ - total traveling time; $u_i$ - generalized forces; $f_{dis}$ - penalty parameter to guarantee free-collision motion ; $\alpha_1$ , $\alpha_2$ , $\alpha_3$ - weighting factors.

Subject to:

$$\max| q_{ji}(t) | \leq QC_j \tag{2}$$

$$\max| \dot{q}_{ji}(t) | \leq VC_j \tag{3}$$

$$\max| \ddot{q}_{ji}(t) | \leq WC_j \tag{4}$$

$$\max| J_{ji}(t) | \leq JC_j \tag{5}$$

$$\max| u_{ji}(t) | \leq UC_j \quad ; \quad \text{for } j=1,2,...,n \quad and \quad i=1,2,...,m-1 \tag{6}$$

$$d_{lq}^0 - d_{lq}(t) \leq 0 \quad \text{ for } (l\,,q) \,\varepsilon\, I_d \tag{7}$$

where: $QC_j$ is the displacement constraint, $VC_j$ is the velocity constraint, $WC_j$ is the acceleration constraint, $JC_j$ is the jerk constraint (jerk is defined as the rate of change of acceleration), and $UC_j$ is the force/torque constraint for joint $j$. The obstacle avoidance is given by $d_{lq}(t)$, where $I_d$ represents the set of possibly colliding pairs of parts.

According to Saramago & Steffen (1998) the generalized forces are calculated as:

$$u_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_j + I_{ai}\ddot{q}_i + \sum_{j=1}^{n}\sum_{k=1}^{j} C_{ijk}\dot{q}_j\dot{q}_k + G_i \quad , \quad i=1,n \tag{8}$$

where:

$$D_{ij} = \sum_{p=maxi,j}^{n} Trace\left[\frac{\partial T_0^p}{\partial q_j} J_p \left(\frac{\partial T_0^p}{\partial q_i}\right)^T\right] \tag{9}$$

$$C_{ijk} = \sum_{p=max(i,j,k)}^{n} Trace\left[\frac{\partial^2 T_0^p}{\partial q_j \partial q_k} U_{pjk} J_p \left(\frac{\partial T_0^p}{\partial q_i}\right)^T\right] \tag{10}$$

$$G_i = \sum_{p=i}^{n} -m_p g^T \left(\frac{\partial T_0^p}{\partial q_i}\overline{r_p}\right) \tag{11}$$

$D_{ij}$: inertia system matrix, $Ia_i$: actuator inertia; $C_{ijk}$: Coriolis and centripetal forces matrix; $J_i$: moments of inertia; $\overline{r}_i$ : center of mass; $G_i$: gravity loading vector; $g$: acceleration due to gravity with respect to the base coordinate system.

Energy dissipation is taken into-account in this paper using the model presented by Van Willigenburg & Loop (1991) in which both friction (Coulomb) and linear viscous damping are considered together to write the dissipation force:

$$F_{diss} = f_c \; sign(\dot{q}) + f_d \; \dot{q} \tag{12}$$

where: $f_c$ is the Coulomb force coefficient and $f_d$ is the viscous damping coefficient .

The generalized forces can be rewritten including the dissipation terms:

$$u_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_j + Ia_i\ddot{q}_i + \sum_{j=1}^{n}\sum_{k=1}^{j} C_{ijk}\dot{q}_j\dot{q}_k + G_i + F_{diss_i} \quad , \quad i=1,n \tag{13}$$

To construct the joint trajectories, given only the initial and final points($q_1$ and $q_m$), as shown in Fig. 1, let a uniform cubic B-spline defined as a polynomial third degree function $f$.

If the terminal time $T$ is fixed, the trajectory $q_i(t)$ will be composed of uniform cubic B-splines with knots $0 = t_1 < t_2 < \cdots < t_{m-1} < t_m = T$, i. e.

$$q_i(t) = \sum_{j=0}^{m-1} q_j^i(t) \, , \quad i=1,...,n \tag{14}$$

where,

$$q_j^i(t) = \gamma_{j-3}^i b_{-3}(t) + \gamma_{j-2}^i b_{-2}(t) + \gamma_{j-1}^i b_{-1}(t) + \gamma_{j-0}^i b_{-0}(t) \qquad (15)$$

$b_{-j}$ are the basis function and $\gamma_j^i$ are the coefficients of the B-spline approximation for $q_i(t)$ on the interval $I_j$.

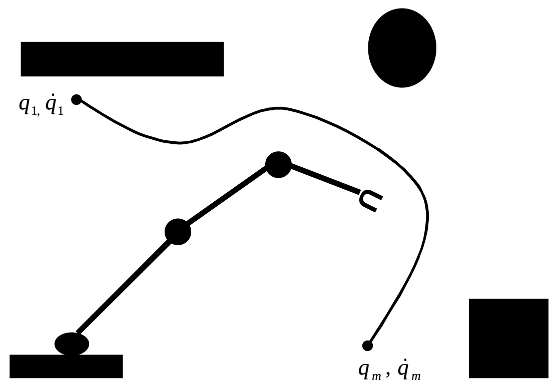


Figure 1 – Trajectory of the end-effector.

For the problems with free terminal time a new time variable $\tau = t/T$ is introduced. This way the interval [0,*T*] is replaced by the non-dimensional interval [0,1]. In this case, the trajectories $q_i(\tau)$ are obtained with knots $0 = \tau_1 < \tau_2 < \cdots < \tau_{m-1} < \tau_m = 1$ as:

$$q_i(\tau) = \sum_{j=0}^{m-1} q_j^i(\tau), \quad i=1,...,n \qquad (16)$$

where $q_j^i(\tau)$ is given by Eq. (15). Since $q_j^i(t)$ is a cubic polynomial in *t*, its derivatives with respect to *t* are well defined . The initial guess for the unknown spline coefficients is obtained
by fitting the straight lines joining the given initial ($q_1$) and final ($q_m$).

When the *m* points and terminal time *T* are adopted, Eq. (16) leads to *n(m+2)* unknowns $\gamma_j^i$. The number of equations for each joint is *m*. So it is necessary, given velocities $\dot{q}_1, \dot{q}_m$, to add two new equations to the linear system. This linear system can be represented by:

$$A\ [\gamma_j^i] = \begin{bmatrix} q_j^i \\ \dot{q}_1^i \\ \dot{q}_m^i \end{bmatrix} \quad j=1,...,m \ \ and \ \ i=1,...,n \qquad (17)$$

Thus, adopting the points $q_i$, the unknown spline coefficients $\gamma_j^i$ are obtained solving system (17), the displacement is obtained using Eq. (15), the velocity and the acceleration are given by derivative of the Eq. (15). These equations are used to determine the inequality kinematic constraints (see Eq. (2)-(5) above). In the optimization of the trajectories the decision variables are the polynomial coefficients $\gamma_j^i$ and the total time *T*. We note that the dimension of this design variable vector is *n(m*+2*)*+1.

## 3. OBSTACLE AVOIDANCE

In this problem, besides the kinematic constraints, we must add special constraints corresponding to obstacle avoidance. These constraints require that:

$$\psi_l(t) \cap \psi_q(t) = \varnothing \quad \text{for } (l, q) \;\varepsilon\; I_d \tag{18}$$

where $\psi_l(t)$ are sets in $R^2$ or $R^3$ and $\varnothing$ is the empty set.

The sets $\psi_l(t)$ describe the space occupied by the parts of the manipulator and the space occupied by the obstacles in the manipulator workspace as given by Eq. (19). The sets $C_l$ characterize the shape of the rigid bodies, while $T_l(t)$ and $R_l(t)$ describe the translation and rotation of the bodies, respectively, i.e.:

$$\psi_l(t) = T_l(t)\, R_l\,(t)\, C_l \tag{19}$$

Each point belonging to an obstacle contour can be calculated as:

$$\begin{bmatrix} x_l(t_i) \\ y_l(t_i) \\ z_l(t_i) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & p_x(t) \\ 0 & 1 & 0 & p_y(t) \\ 0 & 0 & 1 & p_z(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta(t_i)) & -sin(\theta(t_i)) & 0 & 0 \\ sin(\theta(t_i)) & \cos(\theta(t_i)) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_l(t_0) \\ y_l(t_0) \\ z_l(t_0) \\ 1 \end{bmatrix} \tag{20}$$

where $p_x, p_y, p_z$ represent the translation and $\theta(t)$ the rotation of the point $x_l, y_l, z_l$, as shown in Fig. 2. The distance between the sets $d_{lq}(t)$ must be recalculated for all points each time $t_i$ .

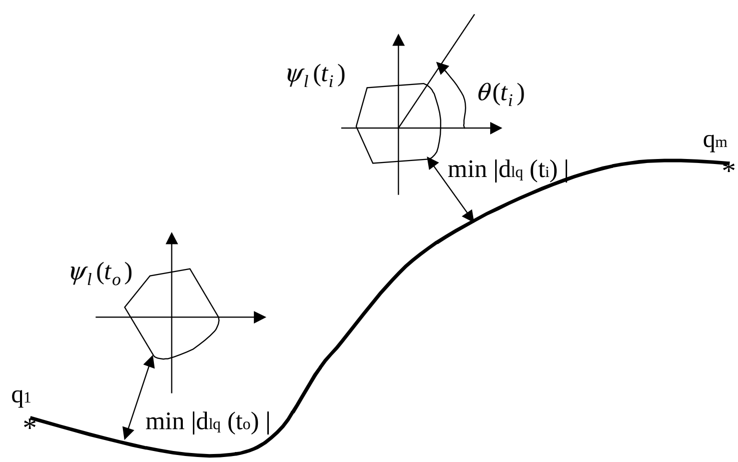


Figure 2 – Moving obstacle representation

It is convenient to describe the requirement (18) by the minimal acceptable distance between the sets:

$$d_{lq}(t) = min\left|z_l - z_q\right| \qquad \text{for } z_l \;\varepsilon\; \psi_l(t),\; z_q \;\varepsilon\; \psi_q(t) \tag{21}$$

To make sure (19) holds with some margin of error, the following conditions are imposed:

$$d_{lq}^{0} - d_{lq}(t) \leq 0 \quad \text{for } (l, q)\ \varepsilon\ I_d \tag{22}$$

where $d_{lq}^{0} > 0$ represents a given tolerance.

The properties of $d_{lq}$ depend on the properties of $C_l$ or $C_q$ . Gilbert & Johnson (1985) proved that, if either $C_l$ or $C_q$ is strictly convex, then $d_{lq}$ is continuously differentiable in the domain.

Equation (22) can be used to represent the obstacle avoidance given by Eq. (7) in the optimal control problem.

When obstacles are found in the workspace it is necessary to add a penalty function to the performance index to guarantee free-collision motion. The idea is to circumscribe each obstacle into a specific sphere. Let $(X_0, Y_0, Z_0)$ be the center of an obstacle and $r_0$ the radius of the sphere that circumscribe this obstacle. The trajectory points $(X, Y, Z)$ which are located outside the sphere are accepted, according to the equation:

$$r_t = \sqrt{(X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2} > r_0 \tag{23}$$

where $r_t$ is the distance between the center of the obstacle and a trajectory point, as represented in Fig. 3.

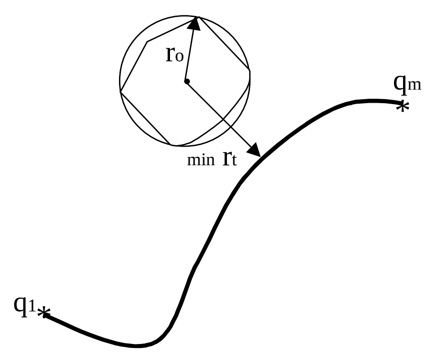


Figure 3 – Obstacle circumscribed by a sphere.

If Eq.(24) is verified the trajectory is out of the sphere and the penalty function ( $f_{dis}$ ) is zero. If the trajectory is tangent or crossing the sphere the performance index will be penalized:

$$\begin{cases} If \quad r_t > r_0 \quad \Rightarrow \quad f_{dis} = 0 \\ If \quad r_t \leq r_0 \quad \Rightarrow \quad f_{dis} = \displaystyle\sum_{l=1}^{n_{obs}} \frac{1}{(\min\ r_t)^2} \end{cases} \tag{24}$$

where $n_{obs}$ is the total number of obstacles in the workspace.

There are situations, according to the topology of the obstacle, where it is more likely to circumscribe the obstacle by an ellipsoid as shown in Fig. 4.

Let $a$, $b$, $c$ be the semi-axes of the circumscribing ellipsoids. Applying the same principle used for the circumscribing spheres, the trajectory points which are located outside the ellipsoid are accepted, according to the equation:

$$r_e = \frac{(X - X_0)^2}{a^2} + \frac{(Y - Y_0)^2}{b^2} + \frac{(Z - Z_0)^2}{c^2} > 1 \tag{25}$$

where $r_e$ is the eccentricity.

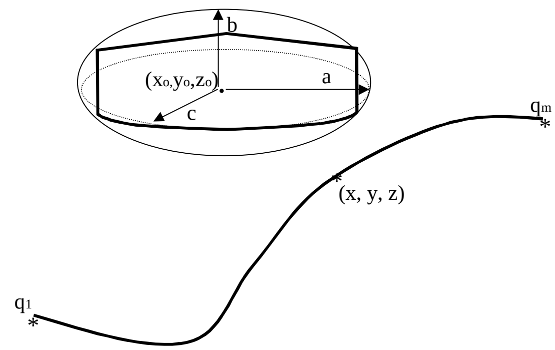


Figure 4 – Obstacle circumscribed by an ellipsoid.

Penalization is used in this case as below:

$$\begin{cases} If \quad r_e > 1 \quad \Rightarrow \quad f_{dis} = 0 \\ If \quad r_e \leq 1 \quad \Rightarrow \quad f_{dis} = \sum_{l=1}^{n_{obs}} \frac{1}{(\min \; r_e)^2} \end{cases} \tag{26}$$

This way the optimal control problem is to minimize the performance index defined by Eq. (1) using the penalty functions given by Eq. (24) or (26), taking into account kinematic and obstacle avoidance constraints.

It is interesting to mention the idea presented by Guldner *et al* (1997) in which an elegant approach for obstacle avoidance is developed using artificial potential fields in such away that the obstacles are protected by spherical or hyper-spherical security zones. To reduce the computational complexity, only the closest obstacles determine the potential field. In the present paper the given methodology uses a very similar principle when penalizing the objective function and circumscribing the obstacle inside spheres or hyper-spheres.

## 4. NUMERICAL APPLICATION

In this paper sequential optimization methods were used. As in this case a multicriterion optimization problem is to be solved a scalar objective function was written using the weighting coefficients method. Using the Augmented Lagrange Multiplier Method a pseudo-objective function is written. Unconstrained minimization is performed by Davidon - Fletcher - Powell method utilizing a combination of the Golden Section and polynomial one-dimensional search procedures. A non-linear optimization code DOT- Design Optimization Tools Program (Vanderplaats, 1995) was coupled to the robot trajectory analysis program in order to obtain an automated design scheme. For the following applications a six d.o.f. Stanford robot manipulator (Paul, 1982) as represented by Fig. 5 and Table 1 was used.

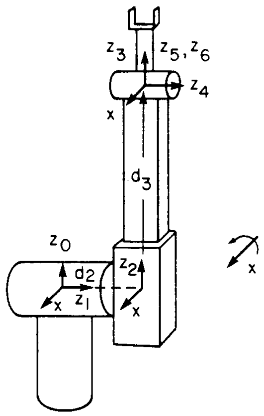


Figure 5 – Stanford robot manipulator.

Table 1 – Link parameters for the Stanford robot

| Joint | $\theta_i$ | $\alpha_i$ | $a_i$ (m) | $d_i$ (m) |
|---|---|---|---|---|
| 1 | $\theta_1$ | $-90^0$ | 0 | 0 |
| 2 | $\theta_2$ | $90^0$ | 0 | 1,2 |
| 3 | 0 | 0 | 0 | $d_3$ |
| 4 | $\theta_4$ | $-90^0$ | 0 | 0 |
| 5 | $\theta_5$ | $90^0$ | 0 | 0 |
| 6 | $\theta_6$ | 0 | 0 | 0 |

It is considered that the robot is initially at rest and comes to a full stop at the end of the trajectory. This way $\dot{q}_1 = \dot{q}_m = \ddot{q}_1 = \ddot{q}_m = 0$ for all joints. Table 2 represents the constraints for displacement, velocities, acceleration, jerk, torque/force. The coefficients associated with dissipation forces ( equation 11) are adopted as: $f_c$ ( *Nm)* = [0,058 0,058 0,058 0,056 0,056 0,056] and $f_d$ ( *Nm/s*)=[0,0005 0,0005 0,000472 0,000382 0,000382 0,000382] .

Table 2 – Constraints for the optimization problem

| Constraint | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| QC (*rd*) | 3,1 | 3,1 | 1,5 | 3,1 | 3,1 | 3,1 |
| VC (*rd/s*) | 2,5 | 2,5 | 2,5 | 2,5 | 2,5 | 2,5 |
| WC ($rd/s^2$) | 9,5 | 9,5 | 9,5 | 9,5 | 9,5 | 9,5 |
| JC ($rd/s^3$) | 50 | 50 | 50 | 50 | 50 | 50 |
| UC (*Nm*) | 50 | 80 | 100 | 10 | 10 | 10 |

Joint 3 – QC (*m*), VC (*m/s*), WC ($m/s^2$), JC ($m/s^3$), UC (*N*)

In this first application one aims at obtaining the end-effector optimal trajectory ($\psi_1$) in the case where the following obstacles are considered: a wall ($\psi_2$), a translating body ($\psi_3$) and a rotating and translating body ($\psi_4$). The initial and final trajectory points are given: $q_1$=[0,1745*rd* 0,1745*rd* 0,80*m* 0,0873*rd* 0,1745*rd* 0,1047*rd*] and $q_m$ = [-1,745*rd* 1,396*rd* 1,20*m* -0,853*rd* 1,078*rd* -1,3894*rd*]. The wall ($\psi_1$) was circumscribed by an ellipsoid and the moving bodies were circumscribed by spheres. The optimization procedure was conducted according to the subsequent steps: the optimal trajectory was firstly obtained for the case in which the obstacles are not taken into account; in the second step the complete problem was solved considering fixed and moving obstacles. The initial total traveling time was *T*=15 *s* and the initial mechanical energy was *E*=42433 *Nm*. The optimal results lead to *T*=12,4 *s* and *E*=27875 *Nm* . Figure 6 shows the end-effector three-dimensional trajectory.

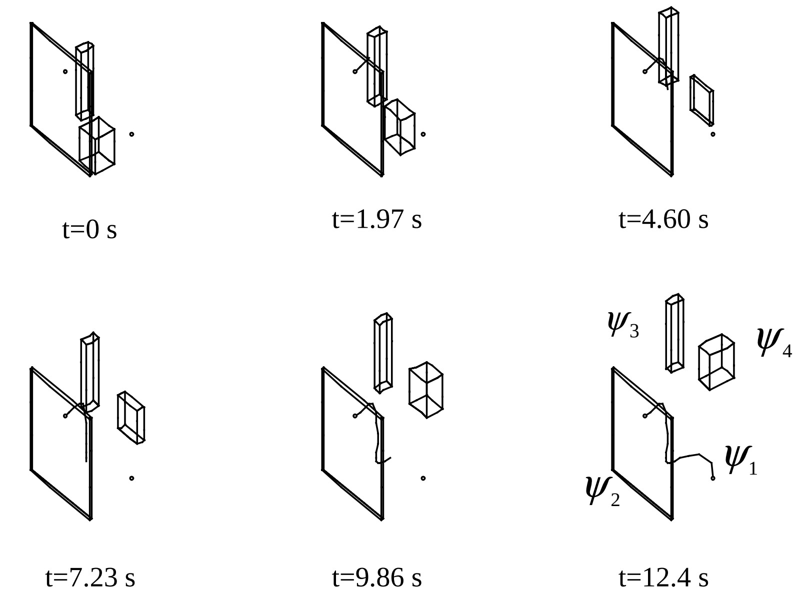


Figure 6 – End-effector three-dimensional optimal trajectory –application 1.

In the second application the end-effector trajectory ($\psi_1$) of a Stanford manipulator has to avoid a pendulum ($\psi_2$) and two fixed obstacles ($\psi_3$ and $\psi_4$). The goal is to obtain the optimal trajectory under the constraints given in Table 2.

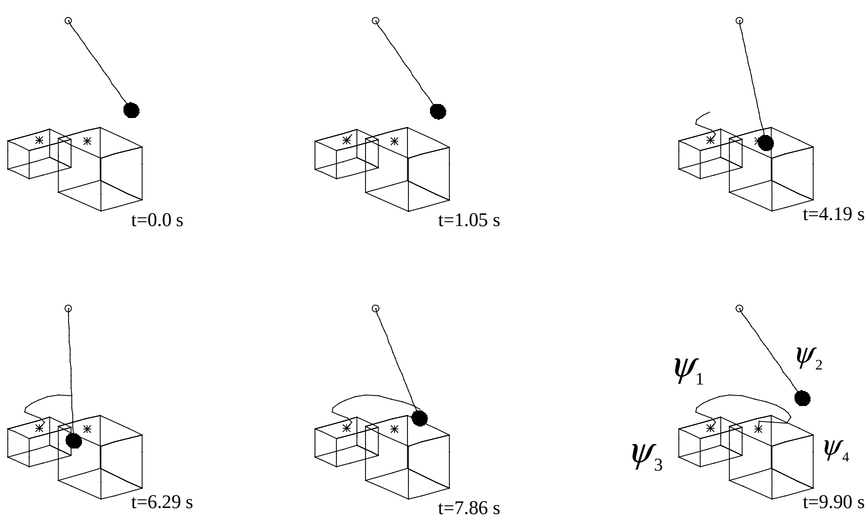


Figure 7 – End-effector three-dimensional optimal trajectory – application 2.

In this case the initial and final points of the end-effector are: $q_1$ = [ 0,1682*rd* 1,3849*rd* 1,1362*m* -0,7555*rd* -0,4702*rd* 0,1472*rd* ] and $q_m$ = [-0,7610*rd* 0,2450*rd* 0,4123*m* 1,4366*rd* 1,0095*rd* –1,0146*rd*]. The initial total traveling time was $T$=11 *s* and the initial mechanical energy was $E$=23462 *Nm*. The optimal results lead to $T$=9,9 *s* and $E$=18871 *Nm* . Figure 7 presents the end-effector three-dimensional trajectory.

## 7. CONCLUSIONS

Obstacle avoidance was obtained by adding penalty functions to the function to be minimized. Besides, constraints which describe minimal acceptable distance between potentially colliding parts are also included in the general non-linear optimization problem. The obstacles are protected by spherical or hyper-spherical security zones which are never penetrated by the end-effector. All second order terms were included in the dynamic equations of motion and friction was considered. In the case of significant operational velocities these terms become important. The manipulator end-effector is represented in the model as a single point. However the methodology can be easily extended to situations in which the geometry of the end-effector is considered. When dealing with moving obstacles either the objective function and the constraint functions have to be up-dated at each time instant. This aspect can be considered a new approach for trajectory optimization problems. The results depend on the weighting coefficients used to formulate the scalar multi-objective function and the initial guess about the design variables.

Two successful numerical applications demonstrated the efficiency of the method. The optimal traveling time and the minimal mechanical energy were obtained in the case where a Stanford manipulator avoids fixed and moving obstacles. It should be pointed out that the solutions obtained are engineering solutions. This means that global minimum is not guaranteed. In the second application significant time reduction is not obtained for free-collision trajectory because the pendulum motion creates additional difficulties with respect to the problem constraints.

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